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PRELIMINARY DESIGN PROCEDURE FOR
END RINGS OF ISOTROPIC CONICAL
SHELLS LOADED BY EXTERNAL PRESSURE

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SUMMARY

The effect of ring size on the buckling of truncated isotropic conical shells supported at the ends by rings and subjected to hydrostatic- or lateral-pressure loadings has been studied. Results were obtained from an approximate theory based on Donnell-type shell theory and a membrane-prestress state. For a given shell geometry, the ring properties were varied to determine the smallest ring required to provide the equivalent of essentially clamped support. For a particular application, this procedure yielded the minimum mass of the shell-ring configuration. A nondimensional stiffness parameter was determined which correlated, in general, the results for the entire range of ring and shell geometries considered. Approximate formulas are presented for the preliminary design of rings with both closed (circular and square) and open (I and Z) cross sections.

INTRODUCTION

Ring stiffeners are often used to provide the edge support for shells of revolution in aerospace applications; for example, ring-supported conical shells and "tension shells" have been proposed for planetary entry vehicles. (See refs. 1 and 2.) Since such vehicles must be of lightweight construction and are exposed to external-pressure and axial loading, a thorough knowledge of the buckling characteristics of such shell-ring configurations is necessary. Recent research on the buckling of ring-supported shells (refs. 1 and 3 to 12) reveals that bifurcation buckling of shells with end rings may occur in either of two modes. If the end rings are not very stiff, the shells buckle almost inextensionally into two circumferential waves at loads which vary considerably with ring stiffness and which can be several orders of magnitude smaller than the buckling loads for clamped shells. (See ref. 4.) If the ring stiffness exceeds some critical value, the shell buckles extensionally into several circumferential waves at loads approaching the buckling loads for clamped ends; these loads do not vary considerably with ring stiffness.

Determination of the smallest (lightest) end rings required to prevent the inextensional form of buckling achieves essentially the maximum buckling load of the shell, generally results in minimum total mass (shell and rings), and thus is an appropriate design goal. In addition, prevention of the inextensional form of buckling allows the design of the shell wall and end rings to be uncoupled. A conservative shell-wall design can be based on simply supported edges. For a given application, the cone angle, end radii, and loading are known, and the required shell stiffness can be obtained from several analyses. (See, for example, refs. 3, 7, 13, and 14.) With the shell-wall properties known, the design problem reduces to the determination of the ring properties required to prevent the inextensional form of buckling.

Although the significant characteristics of shell-ring buckling have been determined, the numerical results available are extremely limited and are completely inadequate for general use in future designs of shell-ring configurations. The lack of numerical data is due primarily to the complexity of the problem and the large number of parameters that significantly affect the buckling characteristics of shell-ring configurations.

The purpose of the present investigation is to provide a quick and reasonably accurate procedure for designing end rings sufficiently stiff to prevent the inextensional form of buckling. Calculated results for truncated isotropic conical shells supported at the ends by rings and subjected to hydrostatic or lateral pressure (large end of shell free of load) are obtained for relatively wide ranges of cone and ring geometries. For a given shell geometry, the ring properties were varied until the smallest ring required to prevent the inextensional form of buckling was determined. These results were obtained by using the approximate theory of reference 7, which utilizes a Donnell-type shell theory and a membrane-prebuckling state. More rigorous analyses are available (refs. 3 and 15) which use numerical techniques that permit use of more exact shell theories and prebuckling states. These analyses, in general, predict smaller ring sizes than the analysis used herein (ref. 7), but they entail more computational effort. Because of the large number of computations required in the present parametric investigation, the reduction in computational effort was considered to justify the loss of accuracy, particularly since the ring-size estimates of the present report are conservative compared with those predicted by more accurate theories.

Numerical results are presented in terms of nondimensional stiffness parameters which correlated the calculations for the entire range of ring and shell geometries considered. Results are obtained for rings with both open and closed cross sections, and formulas which should prove useful for preliminary design of cone-ring configurations are presented for both types of rings.

SYMBOLS

The units for the physical quantities defined in this paper are given both in the U.S. Customary Units and in the International System of Units (SI). Appendix A presents factors relating these two systems of units.

A	cross-sectional area of end ring
B	extensional stiffness; for single-sheet construction, $B = \frac{E_s h}{1 - \mu_s^2}$
b	one-half of the depth of a ring with square, I, or Z cross section
c	characteristic dimension of triangular-ring cross section (see fig. 8)
D	bending stiffness; for single-sheet construction, $D = \frac{E_s h^3}{12(1 - \mu_s^2)}$
E_r	Young's modulus of ring
E_s	Young's modulus of shell
$E_{s,eff}$	effective Young's modulus of shell, $(1 - \mu_s^2)\sqrt{B^3/12D}$
h	wall thickness
h_{eff}	effective wall thickness, $\sqrt{12D/B}$
I_r	moment of inertia of end ring about its centroidal axis parallel to generator of conical shell, $I_r = I_y$
I_y, I_z, I_{yz}	centroidal moments and product of inertia of ring cross section
J	torsional constant of ring cross section
n	circumferential wave number of buckling mode
p	pressure load on shell
R	radius of conical shell (see fig. 1)

r	radius of circular-end-ring cross section
t	thickness of end ring
y, z	orthogonal coordinates (see fig. 1)
z_o	distance of centroid of ring cross section from middle surface of shell, positive for rings on exterior of shell
α	semivertex angle of conical shell
β	ring-eccentricity parameter, $0.19 \left(z_o/r \right)_2$
μ_r	Poisson's ratio of ring
μ_s	Poisson's ratio of shell
ψ	generalized stiffness parameter, $\psi^* \frac{(E_s R h / E_r A)_2^\beta}{(R_2/h)^{0.125}}$
ψ^*	stiffness parameter, $\left(\frac{E_r I_r}{E_s R^{3/2} h^{5/2}} \right)_2$

Subscripts:

f	flange
1	small end of conical shell
2	large end of conical shell

RESULTS AND DISCUSSION

Buckling results have been obtained for the configuration shown in figure 1. The truncated isotropic conical shell is supported by end rings and is subjected to external pressure p . The axial load is either zero (hydrostatic-pressure loading) or adjusted so that the large end of the shell is free of load (lateral-pressure loading). Four types of thin-walled ring cross sections are considered: circular, square, I, and Z. The pertinent dimensions and stiffness properties assumed in this investigation for these cross sections are given in figure 2. Buckling results were obtained from the analysis of

reference 7, which is based on linear Donnell-type shell theory and a membrane prestress state. The governing equations are solved by an assumed displacement method. Up to 40 terms were used to insure convergence of results. Calculations were assumed to be converged when the results for $N + 4$ terms differed by less than 1 percent from the results for N terms. For the buckling calculations, the stiffness of the end rings was taken into account by use of equations (A5) to (A8) of reference 16.

Buckling of Shell-Ring Configurations

Figure 3 shows the effects of end rings on the buckling of a blunt, truncated, sandwich conical shell subjected to lateral pressure. The shell, which was previously studied by Cohen (ref. 1), has end rings consisting of thin-walled circular tubes; the pertinent shell and ring properties are given in figure 4. The results in figure 3 are presented in terms of the pressure p and the radius of the cross section of the ring at the large end of the shell r_2 . The solid curves represent results of the present investigation (obtained from the modal analysis of ref. 7), and the dashed curves were obtained from results presented in reference 1 and calculated from the numerical analysis of reference 15. Results for simply supported and clamped edges from the present investigation are also shown.

When the ring is small, the shell buckles inextensionally into two circumferential waves ($n = 2$), and the buckling load increases rapidly with increasing ring size. When the ring is large, the shell buckles extensionally into seven circumferential waves at a value of the pressure that is bracketed by the results for simply supported and clamped ends. The buckling load in the $n = 7$ mode is influenced only slightly by ring stiffness. Hence, if the ring is large (stiff) enough to suppress the inextensional ($n = 2$) mode of buckling, further increases in ring size would have only a slight effect on the buckling pressure. Thus, the smallest (lightest) ring that will permit exploiting essentially the full buckling capability of the shell is that defined by the intersection points shown by the symbols in figure 3. Note that the value of the required end-ring radius r_2 obtained from the present investigation ($r_2 = 3.4$ inch (8.6 cm)) is approximately 40 percent greater than the value predicted by the results of reference 1; the difference is attributed to the use of a more accurate shell theory and a more accurate prestress state in the calculations of reference 1.

Minimum Mass of Shell-Ring Configurations

As can be seen from figure 3, the inextensional mode of buckling appears undesirable as it can occur for buckling pressures considerably less than the pressures required for extensional buckling. Furthermore, for a shell designed to withstand a given load, an increase in configuration mass generally results if the inextensional mode of buckling is not prevented. For example, figure 5 gives the variation of the masses of the end ring,

shell, and total configuration with r_2 for the shell shown in figure 4, designed to withstand a pressure of 7.2 psi (50 kN/m²).

Calculations were made to determine the required ring size for various values of face-sheet thickness of the sandwich wall. The ratio of the core depth to face-sheet thickness was held constant at a value of 25.

The results in figure 5 for $r_2 = 3.4$ inches (8.6 cm) correspond to the intersection points shown in figure 3 for the shell-wall dimensions given in figure 4. For smaller base rings, a thicker shell wall is required, the shell mass increases, and inextensional buckling governs shell stability. For larger base rings, the shell-wall thickness and mass remain essentially constant at a value approaching the mass of a fully clamped shell (which is the minimum shell mass required to sustain the prescribed load), and extensional buckling governs shell stability. As can be seen, the total-configuration mass is least when r_2 is the minimum value required to prevent the inextensional form of buckling. Thus, the results of figures 3 and 5 indicate that an appropriate goal in the design of shell-ring configurations is the determination of the smallest rings required to prevent the inextensional form of buckling. In addition, prevention of the inextensional form of buckling allows the design of the shell wall and end rings to be uncoupled. For example, a conservative value of the thickness of the shell wall can be obtained from calculations based on simply supported edges. With the shell-wall properties known, the design problem reduces to the determination of the ring properties required to prevent the inextensional form of buckling. All results presented in the remaining sections of this report are ring properties determined at intersection points, such as those shown in figure 3.

Minimum Ring Mass

Since the radii at the ends of the conical shell R_1 and R_2 differ, the required stiffness properties of the two end rings can also be expected to differ. To avoid the inclusion of all of the large number of parameters affecting the buckling of ring-supported shells, it was desirable to establish a definite relationship between the two different end rings. For the shell shown in figure 4, figure 6(a) indicates the effect of changing the ring size at the small end of the shell on the ring size required at the large end. The required value of the radius of the cross section of the ring at the large end of the shell r_2 is plotted as a function of the radius of the cross section of the ring at the small end r_1 .

Results have already been presented in figure 3 for rings of equal thickness ($t_1/t_2 = 1$), whereas on the basis of ring buckling alone, it would appear more realistic to require the ring thickness to be proportional to the shell radius. The results shown in figure 6 were therefore calculated for $t_1/t_2 = R_1/R_2$. As shown in figure 6(a), an

increase in the smaller ring size results in a decrease in the large ring size; the decrease is slight for the ring sizes considered. A desirable combination of ring proportions can be obtained by examining the total mass of the two rings which is shown in figure 6(b). The circular symbol represents the condition $r_1/r_2 = R_1/R_2$, which yields a ring of nearly minimum mass. Thus, the results of figure 6 suggest the use of the design constraints $r_1/r_2 = t_1/t_2 = R_1/R_2$. These constraints and the assumption that both rings are constructed of the same material were used in the general parametric studies of the present investigation.

Shell-Ring Parameters

Because of the large number of parameters affecting the buckling of shell-ring configurations, ground rules were established to limit the number of required calculations. Besides the restrictions that the end rings be of the same material and the design constraints $r_1/r_2 = t_1/t_2 = R_1/R_2$, the calculations were also confined to shells with isotropic bending- and extensional-stiffness properties. Rings with circular, square, I, and Z cross sections were considered. For rings with noncircular cross sections, the radius r_2 is replaced with the half-depth b_2 . The analysis revealed the shell and ring parameters which should be considered. For the general classes of rings considered in this study, several of the basic parameters were related and thus could be replaced by a fewer number of simpler parameters; these basic and simplified parameters are given in the following table:

Parameter	Parametric representation	
	Basic	Simplified
Shell-stiffness ratio	$R_2^2 B/D$	$(R_2/h)^2$
Cone semivertex angle	α	α
Shell-radius ratio	R_1/R_2	R_1/R_2
Poisson's ratio of shell	μ_s	μ_s
Ring-shell extensional-stiffness ratio . . .	$(E_r A/RB)_2$	$(r/R)_2 (E_r t/E_s h)_2$
Poisson's ratio of ring	μ_r	μ_r
Eccentricity of ring centroid	$z_{o,2}$	$(z_o/r)_2$
Ring inertia-area ratios	$\left\{ \begin{array}{l} (I_y/R^2 A)_2 \\ (I_z/R^2 A)_2 \\ (I_{yz}/R^2 A)_2 \end{array} \right.$	$\left. \begin{array}{l} \\ \\ \end{array} \right\} (r/R)_2^2$
Ring torsional-constant—area ratio	$(J/R^2 A)_2$	$(r/R)_2^2$

For single-sheet construction, the shell stiffnesses B and D can be expressed in terms of E_S , h , and μ_S as

$$\left. \begin{aligned} B &= \frac{E_S h}{1 - \mu_S^2} \\ D &= \frac{E_S h^3}{12(1 - \mu_S^2)} \end{aligned} \right\} \quad (1)$$

where E_S is Young's modulus, μ_S Poisson's ratio, and h the shell wall thickness. For general isotropic wall construction, the effective values of E_S and h are defined as

$$\left. \begin{aligned} E_{S,eff} &= (1 - \mu_S^2) \sqrt{B^3/12D} \\ h_{eff} &= \sqrt{12D/B} \end{aligned} \right\} \quad (2)$$

Thus, in any of the parameters used herein E_S and h can be replaced by $E_{S,eff}$ and h_{eff} , respectively.

Calculations were made for the ranges of the various parameters given in the following table:

Parameter	Magnitude	
	Minimum	Maximum
R_2/h	100	1600
α	15°	60°
R_1/R_2	0.2	0.8
$(E_{rt}/E_S h)_2$	0.02	25
$(z_o/r)_2$	0	2

For specified values of these parameters, the ratio $(r/R)_2$ was varied until the smallest value required to prevent the undesirable inextensional form of buckling was determined. Poisson's ratios μ_S and μ_r were taken to be 0.3.

Correlation of Results

The results of reference 10 for ring-supported conical shells indicated that the ring stiffness required to suppress inextensional buckling is related to the buckling pressure by

$$\left(\frac{I_r}{R^4}\right)_2 \propto \frac{p}{E_s} \quad (3)$$

when the cone angle is 60° , the radius ratio R_1/R_2 is $1/3$, and the rings and shells are of the same material. Equation (3) is readily generalized for rings and shells of different materials, which gives

$$\left(\frac{E_r I_r}{E_s R^4}\right)_2 \propto \frac{p}{E_s} \quad (4)$$

A relation between the ratio p/E_s and the shell geometry for extensional buckling can be obtained from an expression developed by considering equivalent simply supported cylinders (ref. 13), which indicates for given values of α and R_1/R_2 that

$$\frac{p}{E_s} \propto \frac{1}{(R_2/h)^{5/2}} \quad (5)$$

Thus for given values of α and R_1/R_2 , equations (4) and (5) indicate that the properties of the end rings required to prevent inextensional buckling are related to the shell-wall properties required to sustain the prescribed load by

$$\left(\frac{E_r I_r}{E_s R^4}\right)_2 \propto \frac{1}{(R_2/h)^{5/2}} \quad (6)$$

or

$$\left(\frac{E_r I_r}{E_s R^{3/2} h^{5/2}}\right)_2 = \psi^* = \text{Constant of proportionality} \quad (7)$$

The stiffness parameter ψ^* is essentially the ratio of the ring-bending stiffness $E_r I_r$ to a combination of the shell-bending and extensional stiffnesses $E_s h^{5/2}$. The results of reference 10 as generalized by equation (7) infer that this ratio may be constant. The more comprehensive results of the present investigation revealed that ψ^* is not, in general, constant but varies with both ring geometry (area and eccentricity) and shell geometry (radius-to-thickness ratio, cone angle, and radius ratio). Thus, a number of calculations were made with systematic variation of these parameters in order to determine the proper modification of ψ^* to obtain a parameter that would be more nearly constant. Details of the modification procedure and the results of the supporting calculations are given in appendix B; the parameter so obtained is the generalized stiffness parameter ψ which is given by

$$\psi = \psi^* \frac{(E_s R h / E_r A)_2^\beta}{(R_2 / h)^{0.125}} = (E_r I_r / E_s R^4)_2 (E_s R^2 / E_r A)_2^\beta (R_2 / h)^{2.375 - \beta} \quad (8)$$

where

$$\beta = 0.19(|z_o / r|)_2 \quad (9)$$

The variation of the stiffness parameter with the cone taper ratio $1 - \frac{R_1}{R_2}$ is shown in figure 7. The open symbols indicate hydrostatic-pressure loading, and the solid symbols indicate lateral-pressure loading. The circular and square symbols in figure 7(a) represent average values of ψ based on numerous calculations for rings of circular and square cross sections, respectively; the solid curve represents an approximate design curve faired through these points. The results for square rings correlate well with the results for circular rings. The higher pressure loading associated with buckling due to lateral pressure did not significantly alter the magnitude of ψ . The variation of ψ with $1 - \frac{R_1}{R_2}$ is approximately linear. The equation for the faired curve in figure 7(a) which gives a linear variation of ψ with $1 - \frac{R_1}{R_2}$ is given by

$$\psi = 0.304 - 0.22 \frac{R_1}{R_2} \quad (10)$$

for a closed cross section. Note that for rings with closed cross sections the value of ψ is independent of cone angle α . However, it is shown in appendix B that for rings with open cross section ψ varies approximately as $1/\cos^4 \alpha$.

The variation of $\psi \cos^4 \alpha$ with the cone taper ratio $1 - \frac{R_1}{R_2}$ is shown in figure 7(b) for rings with open cross sections. Only results for hydrostatic-pressure loading are shown. The circular and square symbols represent average values of $\psi \cos^4 \alpha$ for rings of I and Z cross sections, respectively; the solid line is an approximate design curve faired through these points. The rather large scatter of the results shown in figure 7(b) is attributed to the fact that the moment of inertia I_r is not as representative of the stiffness of rings with open cross sections as it is for rings with symmetric closed cross sections, particularly for large values of α . The results suggest ψ is essentially independent of $1 - \frac{R_1}{R_2}$; the faired curve indicates a constant value of

$$\psi \cos^4 \alpha \approx 0.16 \quad (11)$$

for an open cross section.

Conservative Design Criteria

Equations (10) and (11) indicate the variation of ψ with cone taper ratio based on average values of ψ . To insure conservative designs, equations based on maximum values of ψ should be used. These equations are

$$\psi = 0.367 - 0.27 \frac{R_1}{R_2} \quad (12)$$

for closed cross sections and

$$\psi \cos^4 \alpha = 0.256 \quad (13)$$

for open cross sections.

DESIGN PROCEDURE

The buckling characteristics of ring-supported shells (fig. 3) are such that if inextensional buckling is not permitted, the shell-wall and ring design are not coupled, and the shell-wall design can be based on simply supported edges. For a given application, the cone angle α , end radii R_1 and R_2 , and loading are assumed to be known. The required modulus E_s and thickness h of the shell wall can be obtained from several analyses. (See, for example, refs. 3, 7, and 14.) For hydrostatic loading, the simple algebraic expression based on equivalent cylinder considerations (ref. 13) can be used.

With the shell properties known, the ring properties can be obtained from the parameter ψ in the form (see eqs. (8) and (9))

$$\psi = C \left[\frac{E_r I_r}{(E_r A)^\beta} \right]_2 \quad (14)$$

where

$$C = \frac{(E_s R_2^2)^\beta (R_2/h)^{2.375-\beta}}{E_s R_2^4}$$

For conservative designs, the value of ψ is obtained from equation (12) or (13), and the constant C is known in terms of the ring-eccentricity parameter β since C depends only on the shell properties and β . Thus, the required ring inertia I_r and area A are related by

$$\left(\frac{I_r}{A^\beta} \right)_2 = \frac{\psi}{C (E_r^{1-\beta})_2} \quad (15)$$

At this point the designer is free to specify the ring material, general cross-section shape, and eccentricity of the ring centroid from the middle surface of the shell wall. Also, both the ring inertia and area can be expressed as a function of a characteristic depth and thickness once the cross-section shape is specified. Any combination of depth and thickness that satisfies equation (15) is an acceptable design based solely on prevention of inextensional buckling. The ring properties can be varied to obtain a minimum-mass ring subject to design constraints such as cross-section shape, local buckling or distortion of the ring, minimum gage, and minimum or maximum ring size (radius or depth). Because of the approximate nature of the present calculation, it is recommended that refined calculations based on more rigorous analyses be made on final designs to verify the conservativeness of the present design formulas.

To determine ring buckling precisely, it is necessary to know the stress induced in the ring by the loading on the shell. Accurate ring loads can be obtained only from sophisticated computer programs such as that presented in reference 17. With the loading known, the ring proportions required to prevent ring buckling can be obtained from several analyses. (See, for example, ref. 18.)

Numerical Example

To illustrate the design procedure, the required stiffness properties of rings of triangular cross section will be determined for the sandwich shell shown in figure 4. The triangular cross section shown in figure 8 is a closed cross section, and the design criterion based on results for symmetric closed cross sections (eq. (12)) will be used even though the triangular cross section is not symmetric. The shell properties are

$$R_1/R_2 = 0.445$$

$$\alpha = 60^\circ$$

$$\mu_s = 0.32$$

$$B = 46.7 \times 10^4 \text{ lbf/in. (81.8 MN/m)}$$

$$D = 2.92 \times 10^4 \text{ lbf-in. (3.30 kN-m)}$$

$$R_2 = 90 \text{ inches (229 cm)}$$

This shell geometry was dictated by its particular application and by the external-loading condition. In general, the shell wall would be designed separately, for example, by use of the expression developed for equivalent simply supported cylinders (ref. 13) that relates the hydrostatic-pressure loading to the shell-wall geometry and material properties. From equations (2), $E_{s,eff}$ and h_{eff} are determined to be

$$E_{s,eff} = 48.4 \times 10^4 \text{ psi (3.34 GN/m}^2\text{)}$$

and

$$h_{\text{eff}} = 0.866 \text{ inch (2.2 cm)}$$

and from equation (12) ψ is found to be 0.247. The ring cross section is a 30°, 60°, 90° triangle and is on the inside of the shell. The eccentricity z_0 is -0.167c (fig. 8), and the ring material is assumed to be magnesium. Substituting the appropriate shell and ring properties into equation (15) yields

$$\left(\frac{I_r}{A^{0.0317}} \right)_2 = 18.416 \text{ U.S. Customary Units (722.5 SI Units)} \quad (16)$$

By noting the expressions for $I_{r,2}$ and A_2 in terms of c_2 and t_2 (fig. 8), equation (16) is simplified to

$$c_2^{2.9683} t_2^{0.9683} = 143 \text{ U.S. Customary Units (5611 SI Units)} \quad (17)$$

Thus, the design criterion of equation (12) yields the relation between c_2 and t_2 given by equation (17). The variation of c_2 with t_2 is shown in figure 9. The symbol in figure 9 indicates an actual calculated intersection point, similar to those shown in figure 3; calculations were made for $(c/t)_2 = 25$. As can be seen, the actual calculated value is very close to the value derived from the approximate design criterion and indicates the criterion to be conservative.

Any combination of c_2 and t_2 satisfying equation (17) is a permissible ring design based solely on prevention of inextensional buckling. However, ring mass and design constraints must also be considered. The variation of ring mass with the ring characteristic dimension c_2 is shown in the following table:

c_2		$(c/t)_2$	Mass of ring	
in.	cm		lbm	kg
8	20	9	734	333
10	25	21	463	210
12	30	45	318	144
14	36	84	231	105

These results indicate that the mass decreases with increasing $(c/t)_2$; similar trends for rings of circular cross section were presented in reference 11. Thus, the largest value of $(c/t)_2$ for which the design constraints are not violated gives the minimum-mass ring for the specified ring cross-section shape and material. Curves similar to that in figure 9 for other ring materials, eccentricity, and cross-section shapes can be easily obtained and examined to determine the optimum-ring design. In the present

example, the minimum-mass ring of triangular cross section (preceding table) would have a larger mass than the minimum-mass ring of circular cross section considered previously (fig. 5) if they are compared on the basis of having the same local buckling stress.

CONCLUDING REMARKS

The effect of ring size on the buckling of truncated isotropic conical shells supported at the ends by rings and subjected to hydrostatic or lateral-pressure loading has been studied. Results were obtained from an approximate theory based on Donnell-type shell theory and a membrane-prestress state. For a given shell geometry, the ring properties were varied to determine the smallest ring required to provide the equivalent of essentially clamped edge support. For a particular application, this procedure was shown to yield the minimum mass of the shell-ring configuration. A nondimensional stiffness parameter was obtained which, in general, correlated results for the entire range of ring and shell geometries considered. Formulas which should prove useful for preliminary design of end rings for ring-supported conical shells were presented for rings with both closed and open cross sections.

Langley Research Center,
National Aeronautics and Space Administration,
Hampton, Va., June 18, 1970.

APPENDIX A

CONVERSION OF U.S. CUSTOMARY UNITS TO SI UNITS

The International System of Units (SI) was adopted by the Eleventh General Conference on Weights and Measures, Paris, October 1960 in resolution No. 12 (ref. 19). Conversion factors for the units used herein are given in the following table:

Physical quantity	U.S. Customary Unit	Conversion factor (*)	SI Unit
Area	inch ² (in ²)	0.6452×10^{-3}	meter ² (m ²)
Force	pounds force (lbf)	4.448	newton (N)
Length	inches (in.)	0.0254	meters (m)
Mass	pounds mass (lbm)	0.4536	kilogram (kg)
Moment of inertia	inch ⁴ (in ⁴)	0.4162×10^{-6}	meter ⁴ (m ⁴)
Young's modulus, pressure	$\frac{\text{pounds force}}{\text{inch}^2} \left(\frac{\text{lbf}}{\text{in}^2} \right)$	6.895×10^3	$\frac{\text{newtons}}{\text{meter}^2} \left(\frac{\text{N}}{\text{m}^2} \right)$

*Multiply value given in U.S. Customary Unit by conversion factor to obtain equivalent value in SI Unit.

Prefixes to indicate multiples of units are as follows:

Prefix	Multiple
giga (G)	10^9
mega (M)	10^6
kilo (k)	10^3
centi (c)	10^{-2}
milli (m)	10^{-3}

APPENDIX B

GENERALIZED STIFFNESS PARAMETER

A single parameter ψ that relates for a given radius ratio R_1/R_2 the properties of the end rings required to prevent inextensional buckling to the shell properties required to sustain a prescribed load is derived in this appendix. A first approximation to the desired parameter was obtained from the results of reference 10 and generalized in the text to the form

$$\psi^* = \left(\frac{E_r I_r}{E_s R^{3/2} h^{5/2}} \right)_2 \quad (B1)$$

where ψ^* is essentially the ratio of the ring-bending stiffness $E_r I_r$ to a combination of the shell-bending and extensional stiffnesses $E_s h^{5/2}$. In the following sections, the effects on ψ^* of ring area and eccentricity, of the ratio of shell radius to shell thickness, and of cone angle are considered; and the modifications to ψ^* required to account for these effects are indicated. These modifications lead to the generalized stiffness parameter ψ .

Ring Area and Eccentricity

The variation of ψ^* with the ring-shell extensional-stiffness ratio $(E_r A / E_s R h)_2$ is shown as a log-log plot in figure 10 for rings with circular cross sections. The results are for three values of eccentricity; these values and the range of other specified ring and shell properties are shown in the figure. As can be seen from the figure, ψ^* is independent of ring area (stiffness ratio) for no eccentricity ($z_{o,2} = 0$). For an eccentric ring, the value of ψ^* increases with stiffness ratio but appears to be always limited to less than the value for no eccentricity. The figure also shows that ψ^* is not appreciably affected by changing the position of the ring from inside the shell to outside the shell. For values of the ring-shell extensional-stiffness ratio less than about 0.2, which is the range for practical ring properties, the variation of ψ^* is nearly linear on the log-log plot. The faired curve for ψ^* varies as $(E_r A / E_s R h)_2^{0.19}$ for the ring eccentricity equal to the ring-cross-section radius ($z_{o,2} = \pm r_2$). This variation suggests that the parameter ψ^* should be multiplied by $(E_s R h / E_r A)_2^\beta$ where $\beta = 0.19(|z_o/r|)_2$. The variation of this modified parameter with eccentricity $(|z_o/r|)_2$ is shown in figure 11. The results indicate that $\psi^* (E_s R h / E_r A)_2^\beta$ is essentially independent of $(|z_o/r|)_2$ for values up to at least 2.

APPENDIX B – Continued

Shell Geometry

The variation of $\psi^* (E_s R h / E_r A)_2^\beta$ with the ratio of shell radius to thickness R_2/h is shown as a log-log plot in figure 12 for the range of ring and shell properties given in the figure for rings with circular cross sections. The two symbols for a given value of radius-thickness ratio indicate the maximum and minimum values of the ordinate parameter obtained by varying $\left(\frac{E_r t}{E_s h} \right)_2$. As can be seen, the ordinate variation is slight; by using the faired curve, the ordinate parameter varies approximately as $(R_2/h)^{0.125}$. Thus, the results of figure 12 suggest a further modification which yields the parameter

$$\psi^* \frac{(E_s R h / E_r A)_2^\beta}{(R_2/h)^{0.125}}.$$

The effect of cone angle on this parameter is shown in figure 13, where the variation with $\cos \alpha$ is shown as a log-log plot for a wide range of shell and ring properties. The circular symbols are for rings with circular cross sections, and the square symbols are for rings with I cross sections. The two symbols for a given value of $\cos \alpha$ indicate maximum and minimum values of the modified parameter for the given range of R_2/h and $\left(\frac{E_r t}{E_s h} \right)_2$. As can be seen from figure 13, the modified parameter is essentially independent of $\cos \alpha$ for circular rings but for I-rings varies approximately as $1/\cos^4 \alpha$.

The variation of the modified parameter with α for rings with open cross sections is attributed to the fact that I_r , which is insensitive to variation in α for rings with symmetric closed cross sections, varies significantly with α for rings with open cross sections. The results of figure 13 also reveal considerably more scatter in the results for rings with an I cross section than for those with circular cross sections. Thus, use of the modified parameter results in poorer correlation of results for rings with open cross sections compared with results for rings with closed cross sections.

The results of figures 10 to 13 suggest that the desired generalized stiffness parameter ψ has the form

$$\psi = \psi^* \frac{(E_s R h / E_r A)_2^\beta}{(R_2/h)^{0.125}} \tag{B2}$$

for a closed cross section and

$$\psi \cos^4 \alpha = \psi^* \frac{(E_s R h / E_r A)_2^\beta}{(R_2/h)^{0.125}} \tag{B3}$$

APPENDIX B – Concluded

for an open cross section, where

$$\beta = 0.19(|z_o/r|)_2$$

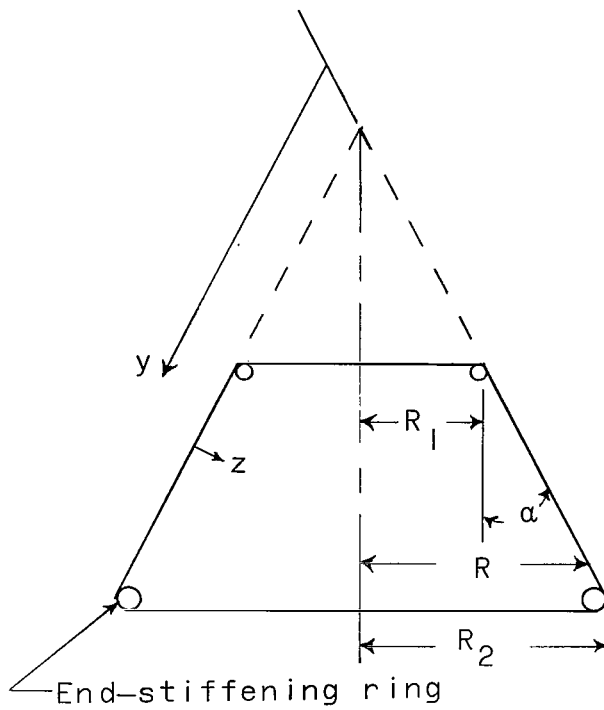
The parameter ψ can be manipulated into the simple parametric form

$$\psi = (E_r I_r / E_s R^4)_2 (E_s R^2 / E_r A)_2^\beta (R_2/h)^{2.375-\beta} \quad (B4)$$

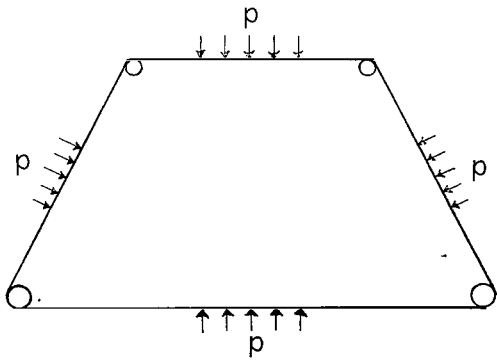
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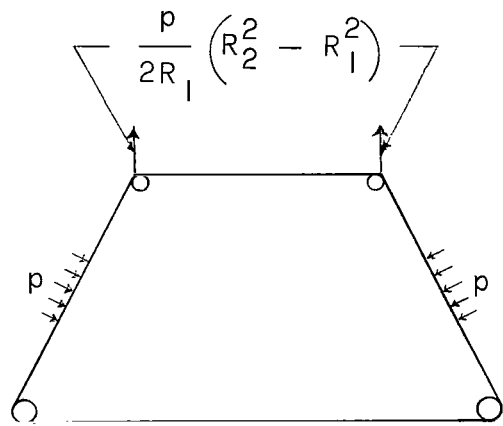
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(a) Configuration and coordinate system.



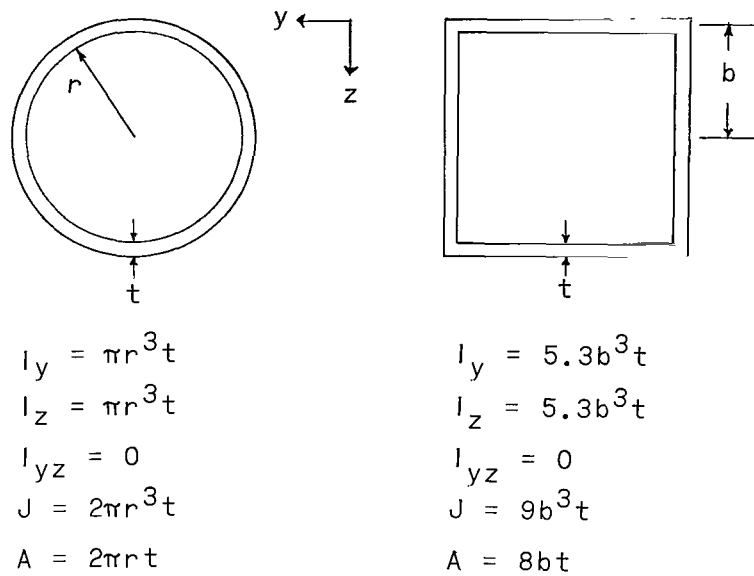
Hydrostatic pressure



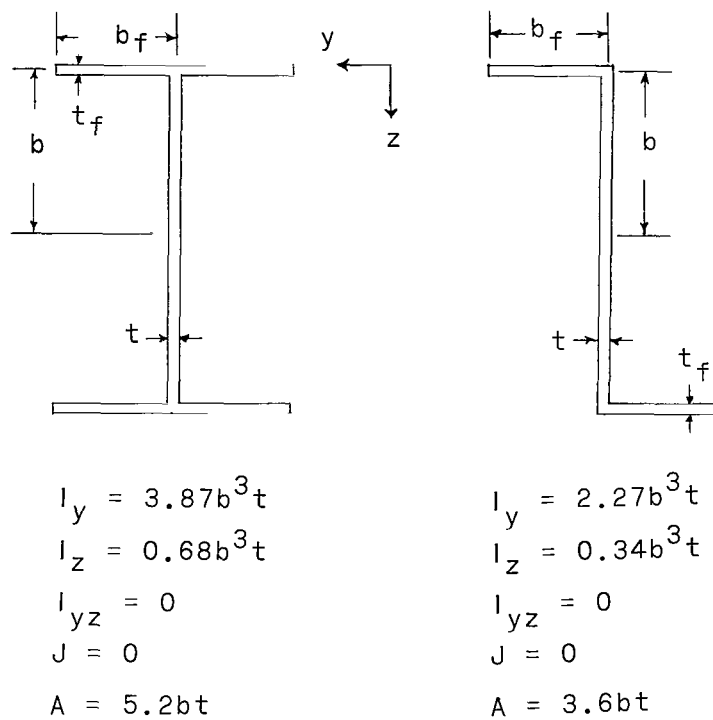
Lateral pressure

(b) Loads.

Figure 1.- Configuration, coordinate system, and loads.



(a) Closed cross sections.



(b) Open cross sections. $t_f/t = 1$; $b_f/b = 0.8$.

Figure 2.- Properties of end-stiffening rings.

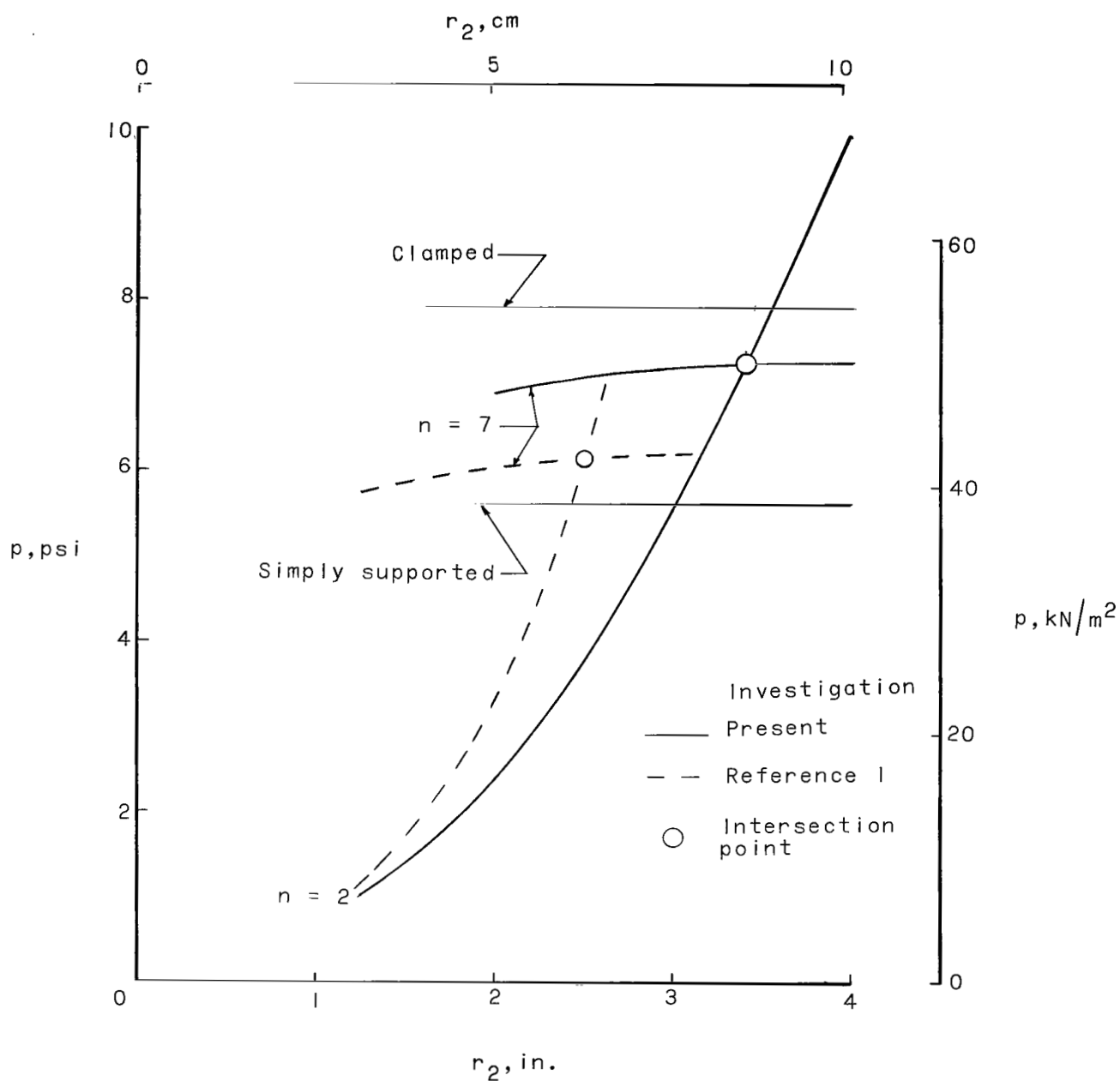


Figure 3.- Effects of end rings with circular cross sections on buckling of blunt, truncated, sandwich conical shell shown in figure 4 subjected to lateral pressure.

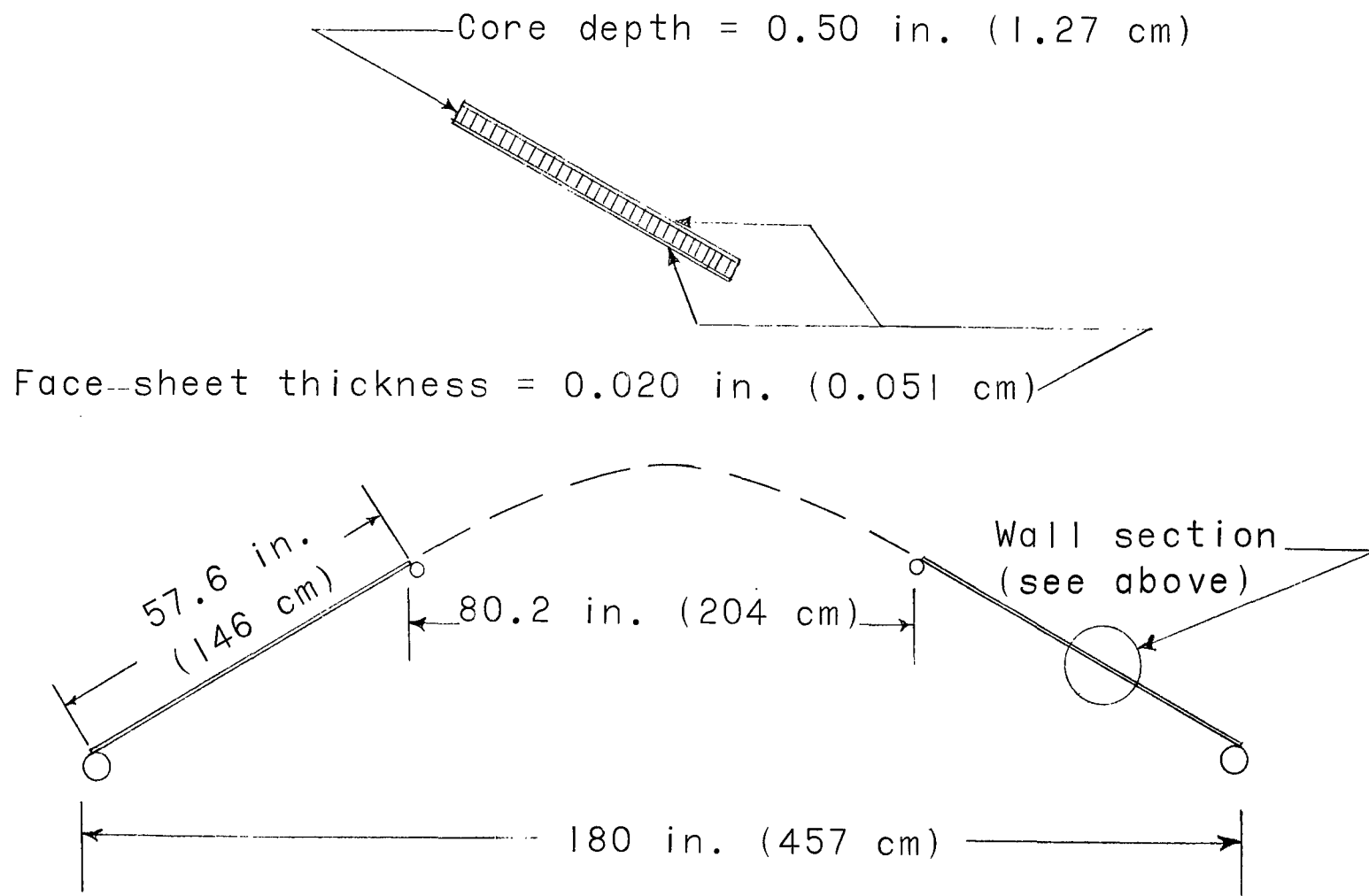


Figure 4.- Cone-buckling model. (See ref. 1.) $R_1/R_2 = 0.445$; $R_2/h_{\text{eff}} = 104$; $\alpha = 60^\circ$; $(r/R)_1 = 0.0312$; $(t/R)_2 = 0.00139$; $t_1/t_2 = 1$.

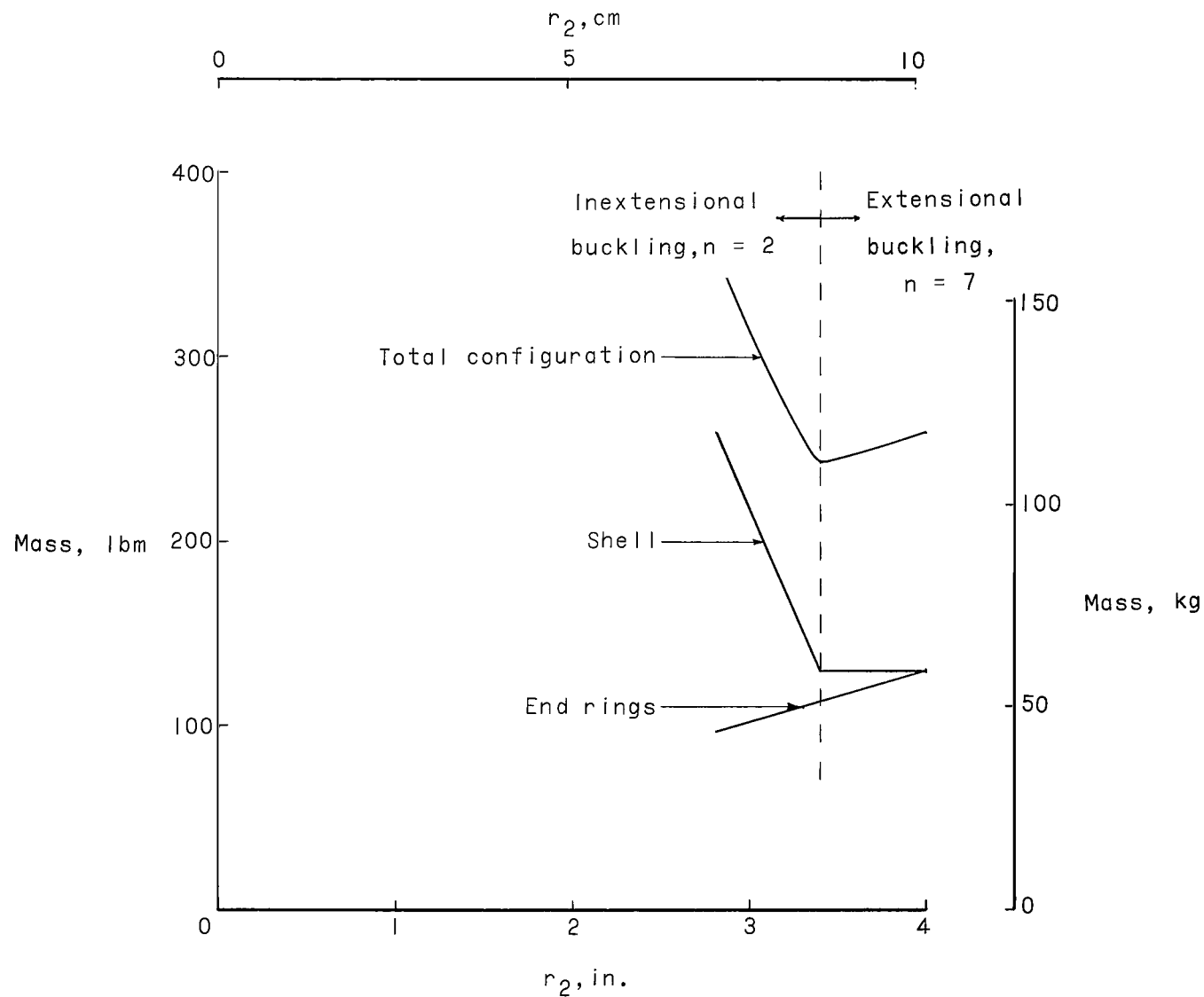
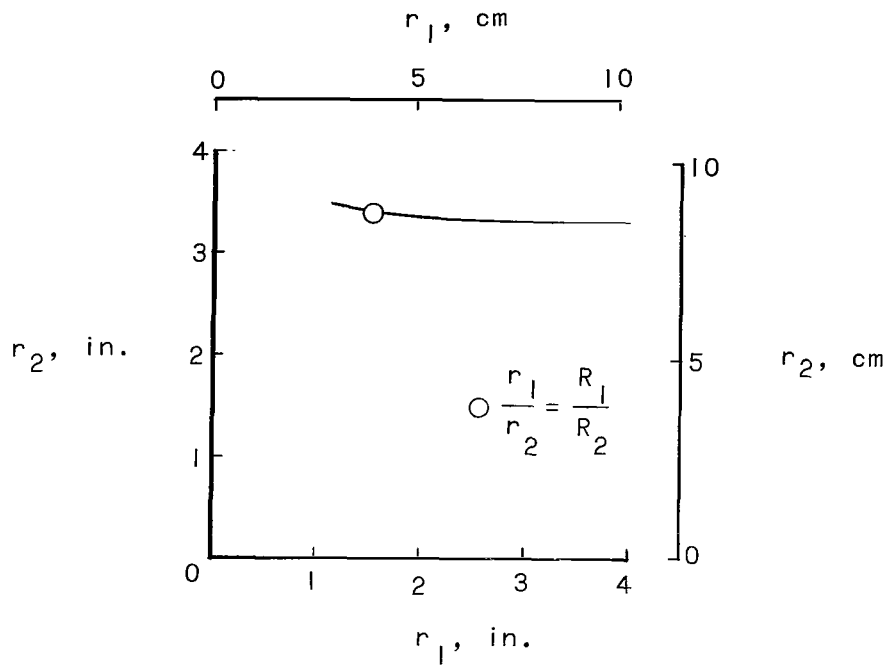
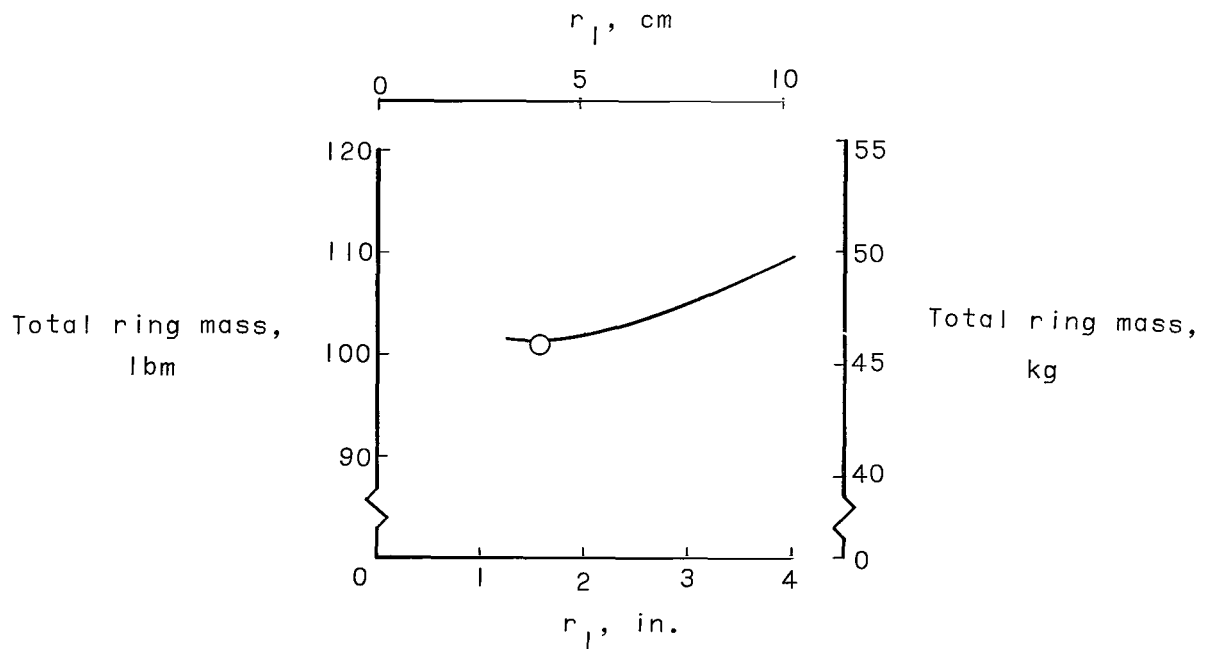


Figure 5.- Effects of end rings of circular cross section on mass of sandwich conical shell subjected to a prescribed lateral pressure. $R_1/R_2 = 0.445$; $57 \leq (R_2/h_{eff}) \leq 104$; $\alpha = 60^\circ$; $(r/R)_1 = 0.0312$; $(t/R)_2 = 0.00139$; $t_1/t_2 = 1$; $R_2 = 90$ in. (229 cm); $p = 7.2$ psi (50 kN/m²); shell-core density, 3 percent.

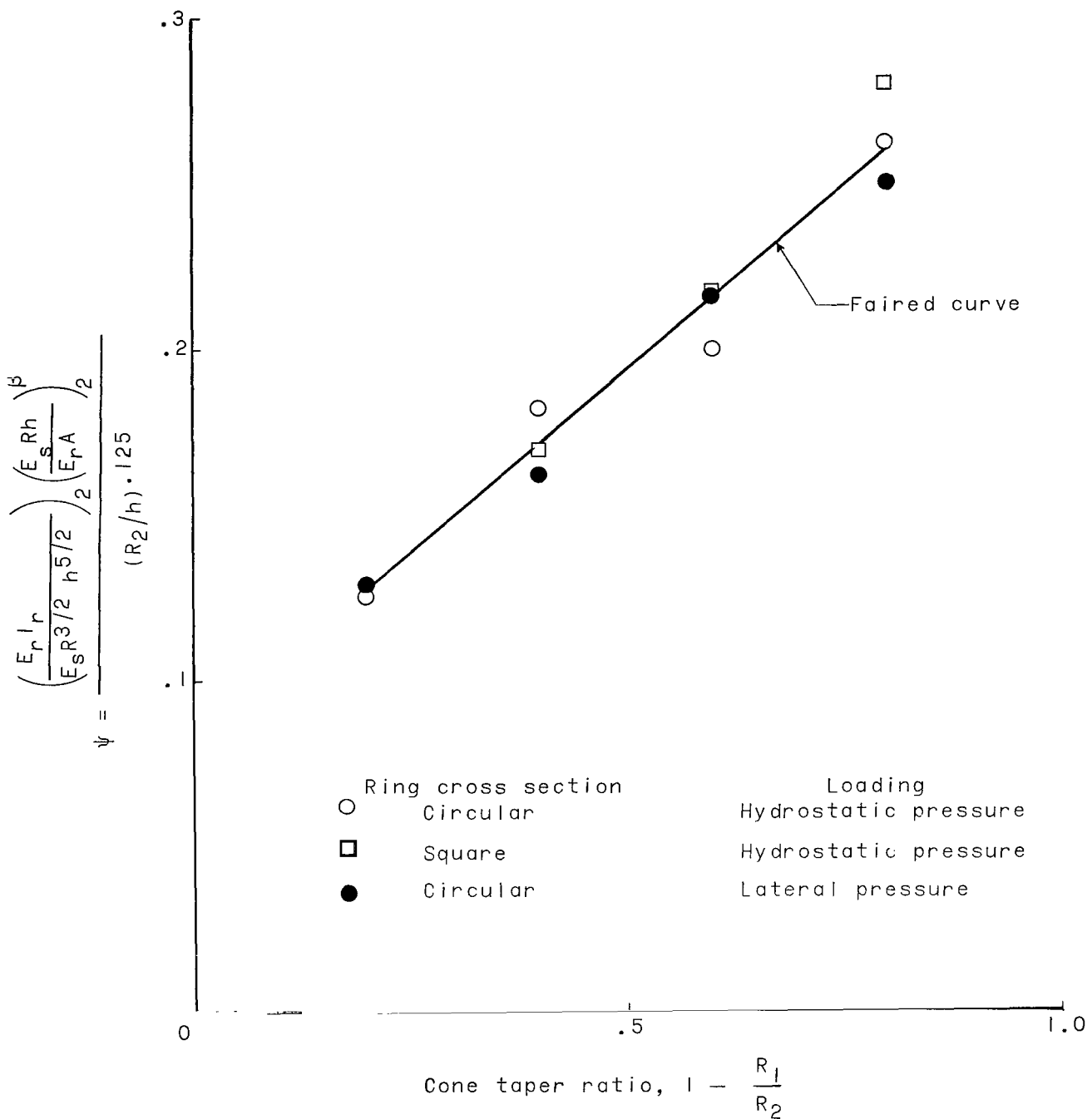


(a) Variation of ring cross-section radius, r_2 .



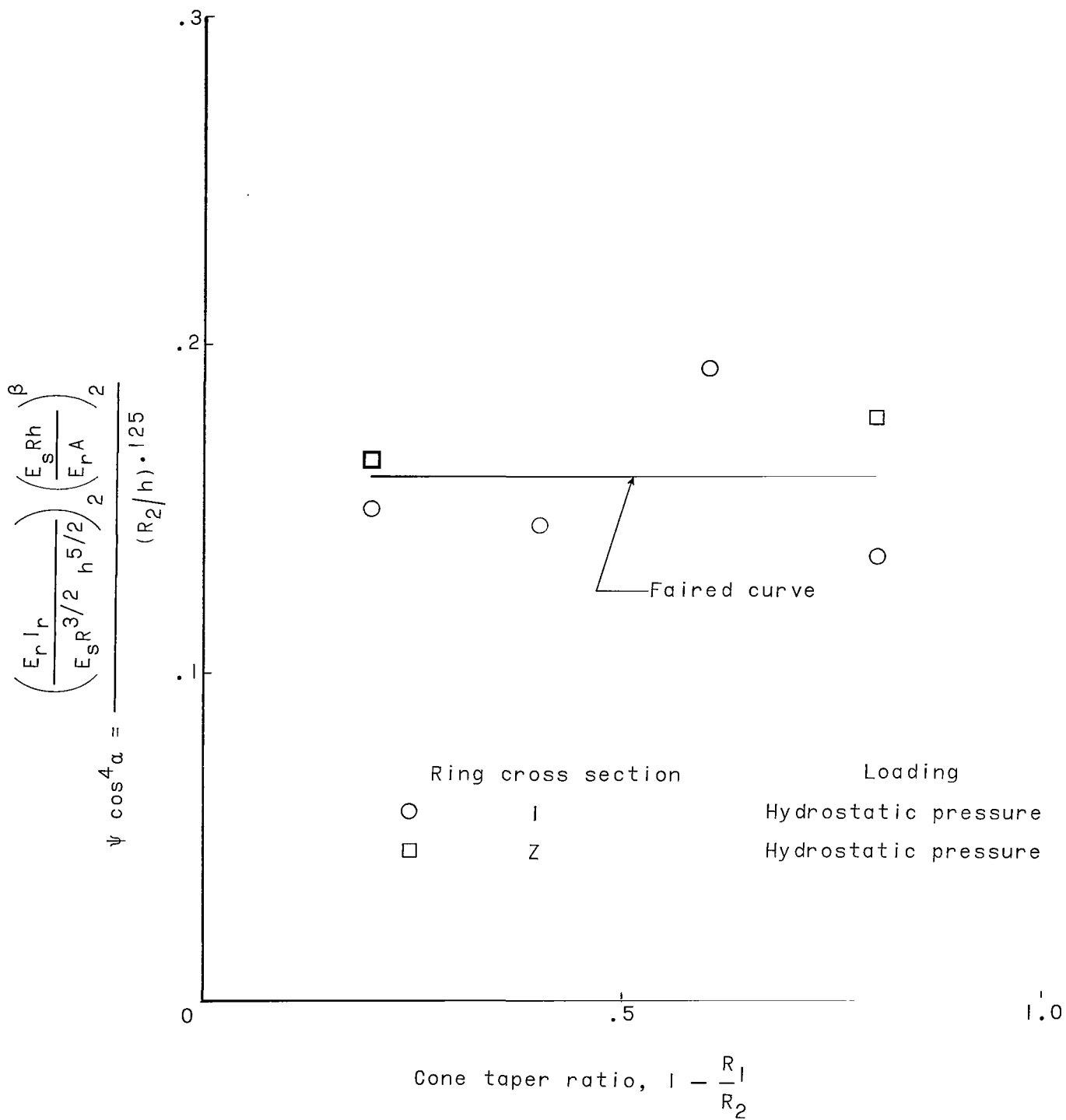
(b) Variation of total ring mass.

Figure 6.- Variation of ring cross-section radius r_2 and total ring mass with ring cross-section radius r_1 for shell shown in figure 4. $t_1/t_2 = R_1/R_2$; $(t/R)_2 = 0.00139$.



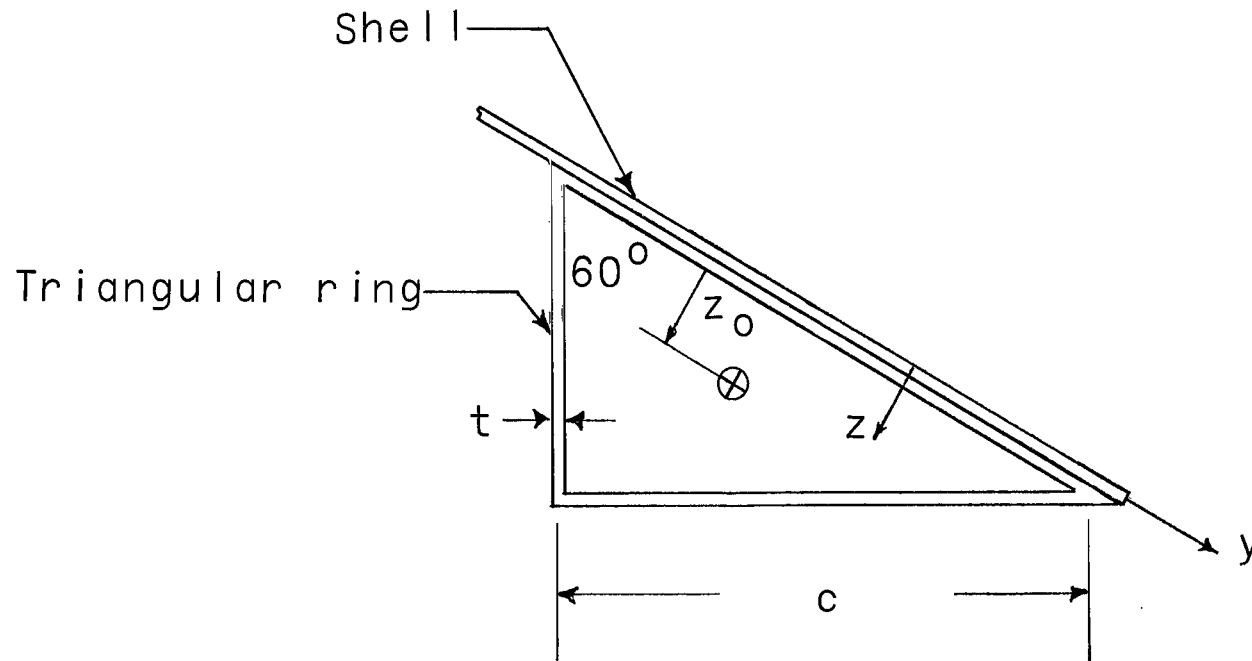
(a) Rings with closed cross sections.

Figure 7.- Variation of average value of stiffness parameter with cone taper ratio. $15^\circ \leq \alpha \leq 60^\circ$; $0.04 \leq \frac{E_r t}{E_s h} \leq 5$;
 $100 \leq (R_2/h) \leq 1600$; $\beta = 0.19 \left(\frac{z_0/r}{2} \right)^2$.



(b) Rings with open cross sections.

Figure 7.- Concluded.



$$I_y = 0.0426c^3t$$

$$I_z = .1758c^3t$$

$$I_{yz} = -.0118c^3t$$

$$J = 1.225c^3t$$

$$A = 2.733ct$$

$$z_0 = -.167c$$

Figure 8.- Properties of triangular ring considered in numerical example.

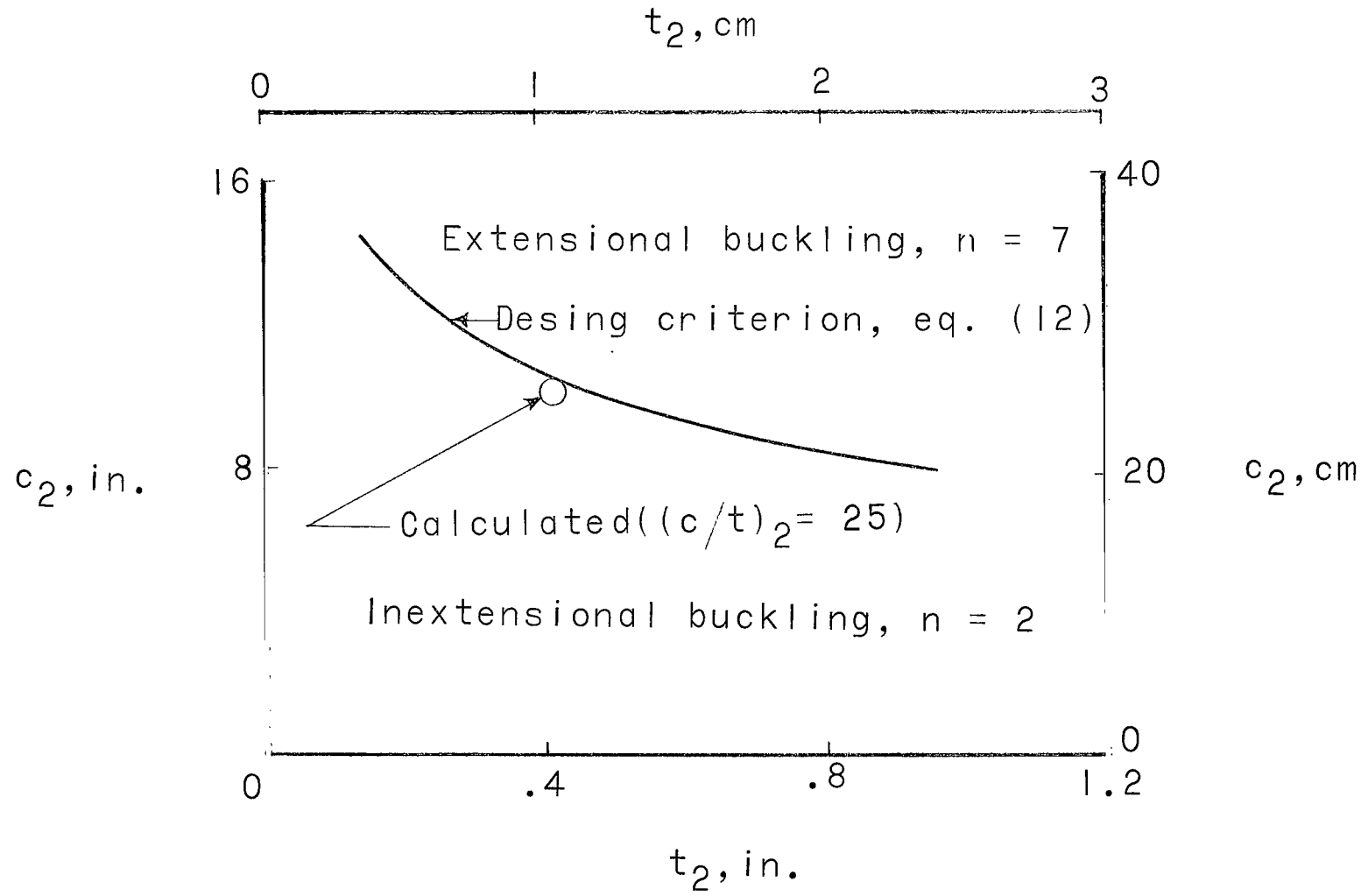


Figure 9.- Variation of characteristic dimension c_2 with thickness required to prevent inextensional buckling t_2 for triangular ring considered in numerical example.

$$\psi^* = \left(\frac{E_r I_r}{E_s R^{3/2} h^{5/2}} \right)_2$$

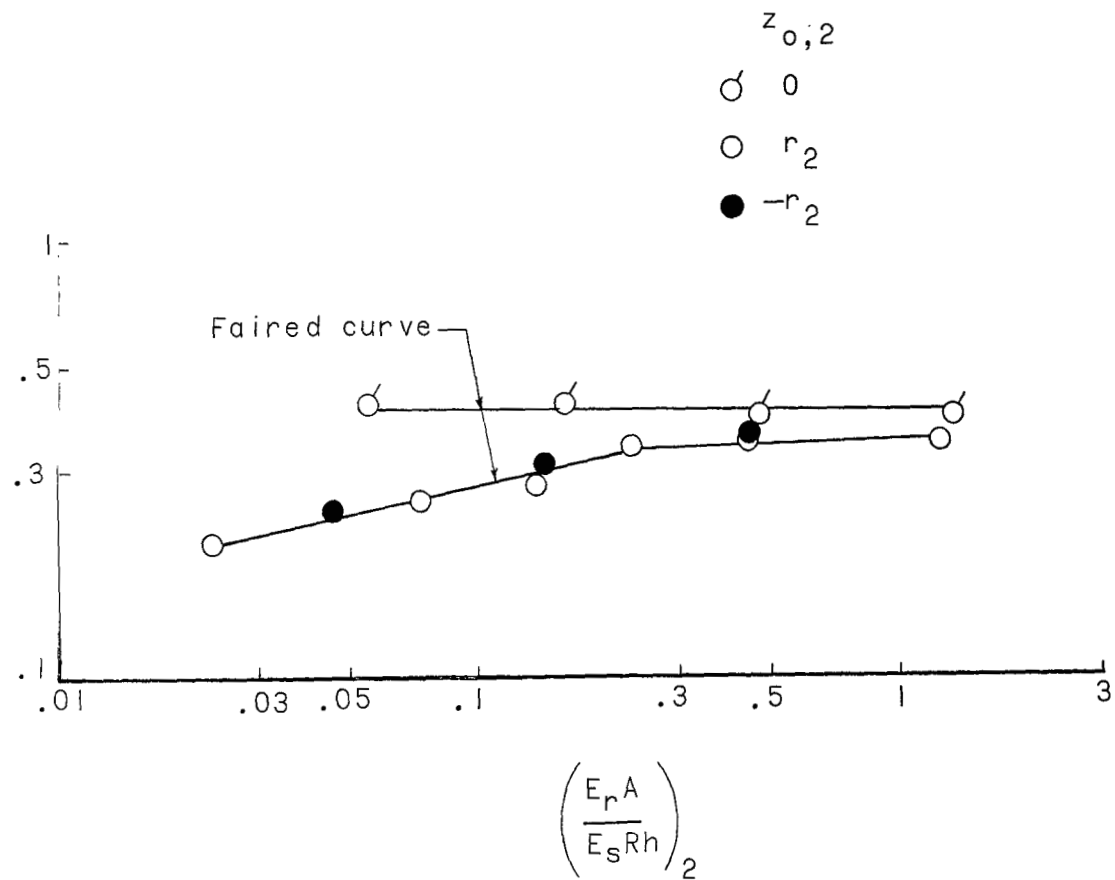


Figure 10.- Variation of ψ^* with $\left(\frac{E_r A}{E_s R h} \right)_2$ for rings with circular cross sections. $R_1/R_2 = 0.4$; $\alpha = 60^\circ$; $0.08 \leq \left(\frac{E_r t}{E_s h} \right)_2 \leq 25$; $R_2/h = 400$.

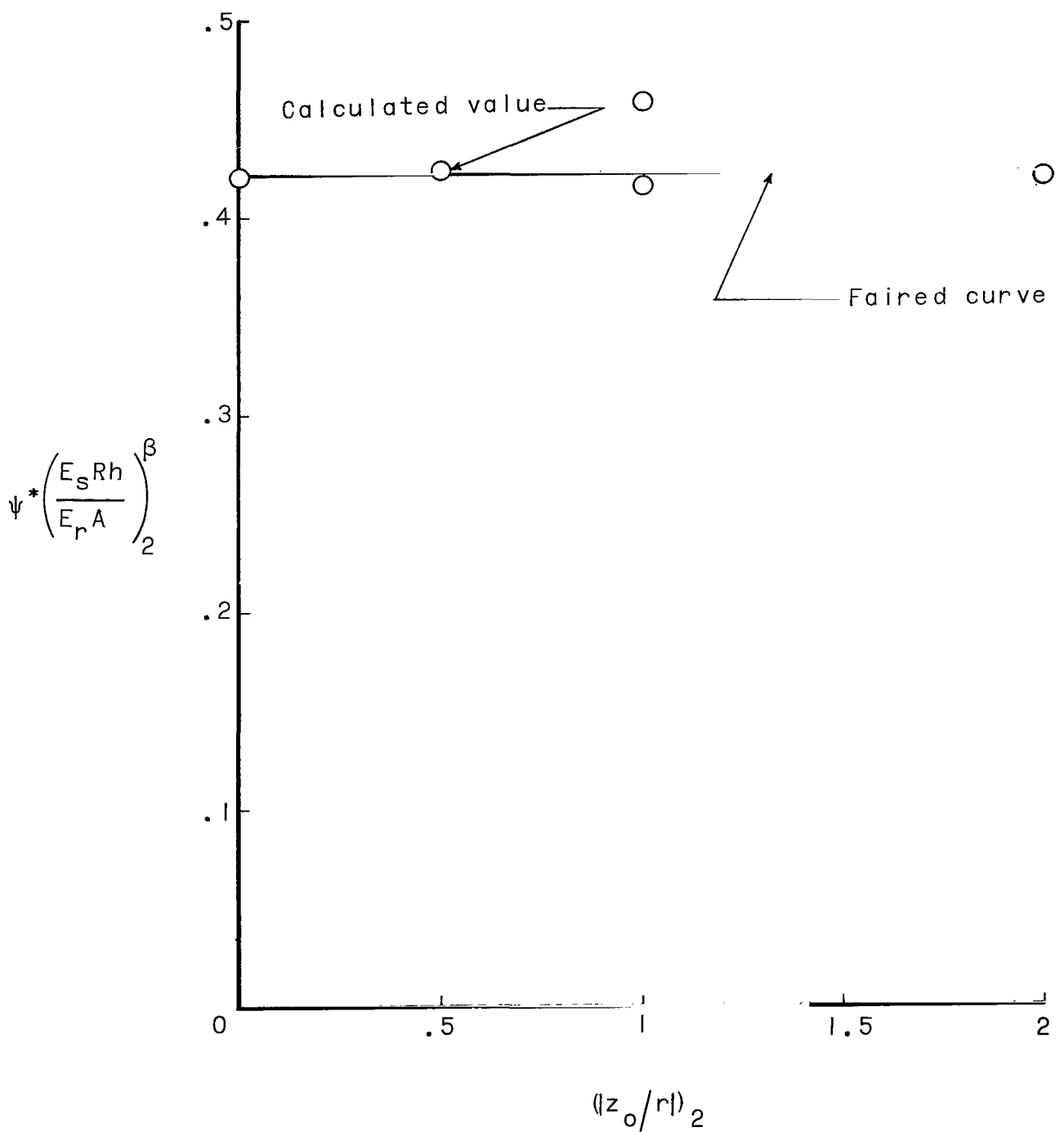


Figure 11.- Variation of $\psi^* \left(\frac{E_s R h}{E_r A} \right)_2^\beta$ with ratio of ring eccentricity to ring radius for rings with circular cross section. $R_1/R_2 = 0.4$;

$$R_2/h = 400; \left(\frac{E_r t}{E_s h} \right)_2 = 1; \alpha = 60^\circ; \beta = 0.19 (|z_0/r|)_2.$$

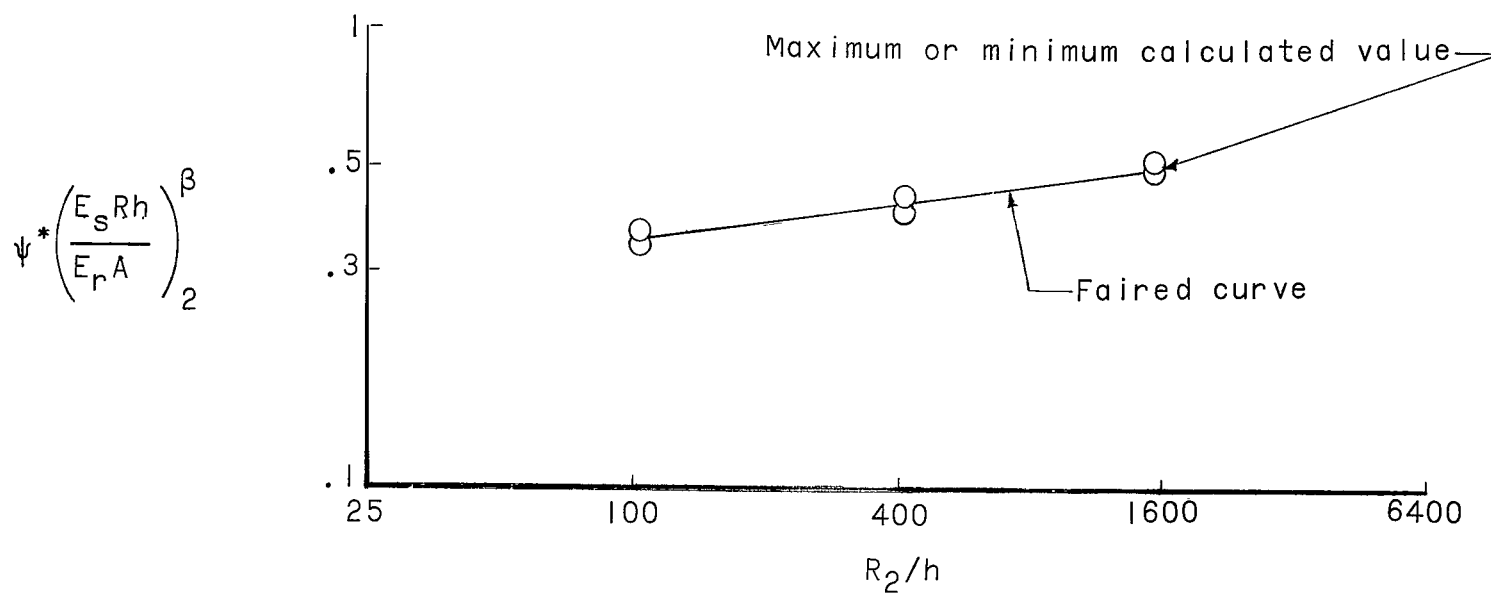


Figure 12.- Variation of $\psi^* \left(\frac{E_s R h}{E_r A} \right)_2^\beta$ with ratio of shell radius to thickness for rings with circular cross sections. $R_1/R_2 = 0.4$; $\alpha = 60^\circ$;

$$0.08 \leq \left(\frac{E_r t}{E_s h} \right)_2 \leq 5; \quad (|z_0/r|)_2 = 1; \quad \beta = 0.19 (|z_0/r|)_2.$$

Generalized stiffness parameter,

$$\frac{\left(\frac{E_r I_r}{E_s R^{3/2} h^{5/2}} \right)^2 \left(\frac{E_s R h}{E_r A} \right)^2}{(R_2/h)^{.125}}$$

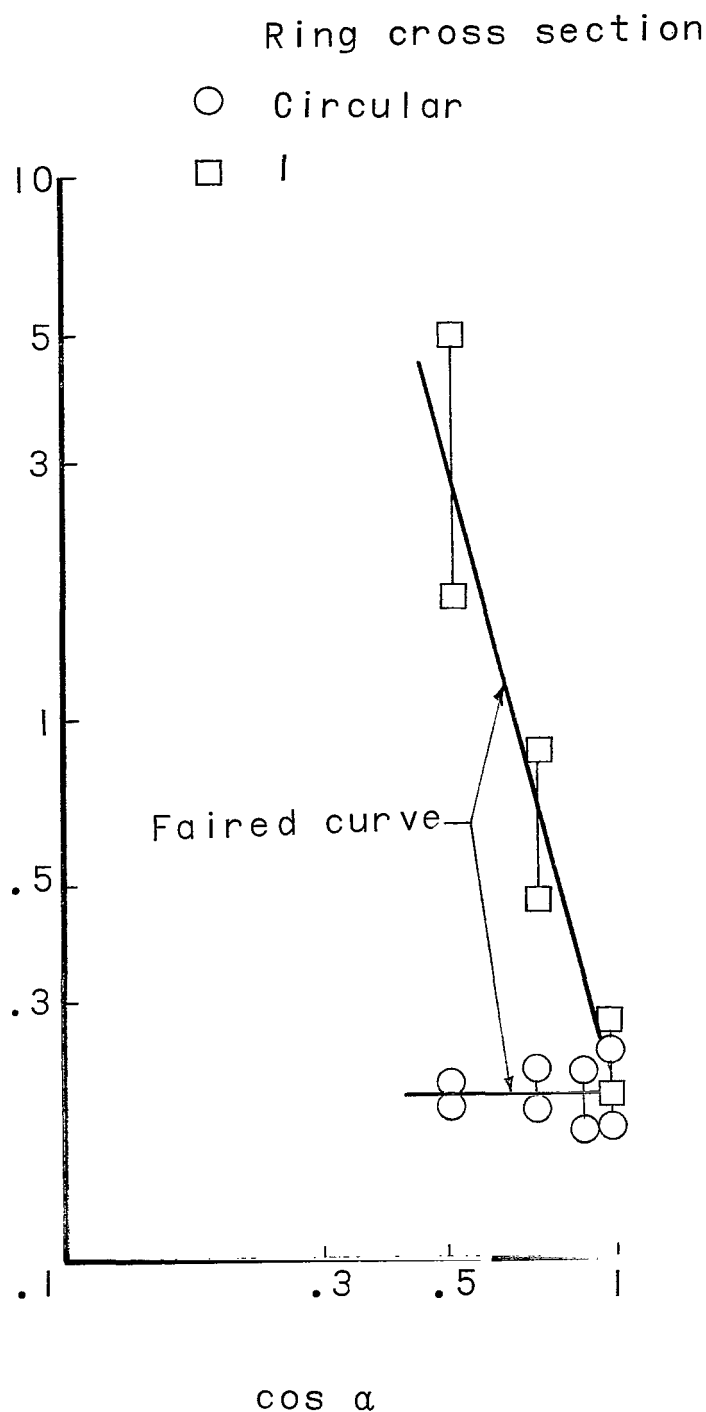


Figure 13.- Variation of stiffness parameter ψ with $\cos \alpha$ for rings with open and closed cross sections. $R_1/R_2 = 0.4$;

$$100 \leq (R_2/h) \leq 1600; 0.08 \leq \left(\frac{E_r t}{E_s h} \right)_2 \leq 5; (z_0/r)_2 = 1.$$



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